Basics of programming language: An introduction using ARLA

Exercises

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Exercise 1. Write an algorithm that reads three real numbers x, y, z and compute the maximum among them.

Exercise 2. Write an algorithm that reads four real numbers x, y, q, z, compute (i) maximum $a_{\max}$ and minimum $a_{\min}$ among them, (ii) the difference $a = a_{\max} - a_{\min}$, and verifies whether $a > 0$.

Exercise 3. Write an algorithm that reads three real numbers x, y, z and computes the value of the following expression:

$$w = \frac{\min(x, y)}{z^2} + \max(x, z)$$

<u>TIPS</u>: min() and max() are not primitive functions in this exercise and need to be written by scratch.

Exercise 4. Write an algorithm that reads two real numbers x and y and execute the following instructions:

1. if $\lfloor \min(x, y) \rfloor$ is an even number then evaluates the expression $z = x^{|y|}$
2. if $\lfloor \min(x, y) \rfloor$ is an odd number then evaluates the expression $z = x^{|y-x|}$
3. outputs the value of z

<u>TIPS</u>: The value of $\lfloor a \rfloor$ can be computed using the primitive function `floor()`, the absolute value $|a|$ can also be computed by means of the primitive `abs()`, whereas to verify whether a number is odd or even, one can use the primitive `mod(a,2)` which gives as output `True` if the number is even.

Exercise 5. Write an algorithm that reads three integers x, y, z and evaluates the following instructions:

1. computes $q = x + y + z$ and, if $q < 0$, transforms the input such that q is always greater than zero
2. if $q < \min(x, y, z)$ then computes $z = q^2 + \frac{1}{2} q^3$
3. if $q > \min(x, y, z)$ then computes $z = \frac{q^2}{2} + \frac{1}{q^3}$

4. outputs z

Exercise 6. Write an algorithm that reads two real numbers x and y and computes the following expression:

$$z = \max(|x|, |y|) - \min(x, y) + (x - y)^2$$

If $z < 0$ then asks for a third number q and evaluates whether $z < q$.

Exercise 7. Write an algorithm that reads a real number a, determines its sign $s = \text{sign}(a)$, and outputs a boolean value $\{1, 0\}$ indicating its sign s.

Exercise 8. Write an algorithm that reads N numbers, computes their sum s until the condition $s > q$ is met.
TIPS: q is a constant stored before the execution of the sum-loop.

Exercise 9. Write an algorithms that reads two intervals $i_1 = [a, b]$ and $i_2 = [c, d]$ and evaluates the following instructions:

- if $i_1 \subset i_2$ then computes $h = (\max(a, b) - \min(a, b)) \frac{1}{j}$, where $j = \frac{a+b}{2}$
- if $i_1 \not\subset i_2$ then computes $h = (\max(a, b) - \min(a, b)) \frac{1}{j}$, where $j = \frac{a}{|b|+\epsilon}$ and $\epsilon = \max(a, b, c, d)^{\frac{1}{3}}$
- if $i_1 = i_2$ then computes $h = (\max(a, b) - \min(a, b))$
- outputs h

Exercise 10. Write an algorithm that reads two points on $\mathbb{R}^2$, i.e. $\mathbf{x} = (x_1, y_1)$ and $\mathbf{y} = (x_2, y_2)$ and determines if the line passing through these points also passes through the origin $\mathbf{c} = (0, 0)$ or not.
TIPS: To check for the line passing through the origin, the equation $x_1(y_2 - y_1) = y_1(x_2 - x_1)$ must be satisfied.

Exercise 11. Write an algorithm that reads a $I \times 1$ vector of real numbers $\mathbf{x} = (x_1, \ldots, x_i, \ldots, x_I)$ and gives as output the vector $\mathbf{y} = (x_I, \ldots, x_i, \ldots, x_1)$.

Exercise 12. Write an algorithm that reads two natural numbers n_a and n_b and determines the vector $\mathbf{x}$ containing the sequence of numbers included between n_a and n_b.

Exercise 13. Write an algorithm that reads the array $\mathbf{x}_{n\times 1}$ of real numbers and evaluates the following expressions:

1. $x^* = \sum_{i=1}^{n} x_i$
2. $\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$
3. $x^\dagger = x^* / (\max(\mathbf{x}) - \min(\mathbf{x}))$
4. $z = (x^* - 2x^\dagger)$
5. write as output z

Exercise 14. Write an algorithm that reads a vector of integers $\mathbf{n}_{I\times 1}$ and computes the vector $\mathbf{f}_{I\times 1}$ containing the frequencies of the elements of $\mathbf{n}_{I\times 1}$.
TIPS: Write a function that counts how many times the elements of $\mathbf{n}$ occur.

Exercise 15. (*cont. Exercise 14*) Write an algorithm that reads as input the vector of frequencies $\mathbf{f}_{I\times 1}$ and computes the vector $\mathbf{s}_{I\times 1}$ where the generic element s_i is defined as follows:

$$s_i = s_{i-1} + f_i$$

Note that $s_1 = f_1$ by definition.

Exercise 16. Write an algorithms that reads two vectors of reals $\mathbf{x}_{n\times 1}$ and $\mathbf{s}_{n\times 1}$ and computes the following expression:

$$v_i = x_i + (x_{i+1} - x_i)\frac{0.5 - s_i}{s_{i+1} - s_i}$$

TIPS: The vector $\mathbf{s}$ contains the cumulative frequencies of $\mathbf{x}$.

Exercise 17. Write an algorithm to approximate the computation of following expression:

$$\int_1^N z^2 + \sin(z)dz \;\approx\; \frac{N-1}{N}\sum_{i=1}^{N} z_i^2 + \sin(z_i)$$

given a vector of reals $\mathbf{z}_{N\times 1}$ as input.

Exercise 18. Write an algorithm that compute the square root of a number $s > 0$ using the Heron's approximation:

$$\sqrt{s} = \lim_{n\to\infty} x_n$$

where the generic element x_n is defined as:

$$x_n = \frac{1}{2}\left(x_{n-1} + \frac{s}{x_{n-1}}\right)$$

with $x_1 = 1.5$. The iterative approximation must be executed until the condition $(x_n - x_{n-1}) < \epsilon$ is met, with ϵ being a user's defined small real number (e.g., $\epsilon = 0.001$).

Exercise 19. Write an algorithm that reads two vectors $\mathbf{x}_{n\times 1}$ and $\mathbf{y}_{n\times 1}$ containing pairs of coordinates (integers), and computes the Manhattan distance between all pairs of points stored in the input vectors:

$$d = \sum_{i=1}^{n}\sum_{j=i+1}^{n} |x_i - x_j| + |y_i - y_j|$$

TIPS: Use a brute-force approach.

Exercise 20. Write an algorithm that reads a vector of strings $\mathbf{t}_{n\times 1}$ and a single string s, and verifies if s is contained in $\mathbf{t}_{n\times 1}$.

Exercise 21. Write an algorithm that reads a $N \times M$ matrix $\mathbf{X}$ with $N = M$ and computes the following quantities:

1. $q = \sum_{i=1}^{N} \text{diag}(\mathbf{X})_i + \max(\mathbf{X})$
2. $p = \prod_{i=1}^{N} \text{diag}(\mathbf{X})_i + \min(\mathbf{X})$
3. if $q > p$ outputs p, otherwise outputs q

Exercise 22. Write an algorithm that reads a $N \times M$ matrix $\mathbf{X}$ with $N = M$, transforms the input array into:

$$\mathbf{x} = (X_{1,1}, \ldots, X_{1,M}, \ldots, X_{i,M}, \ldots, X_{N,M})$$

and outputs $\mathbf{x}$.

Exercise 23. Write an algorithm that reads a $I \times J$ matrix of reals $\mathbf{A}$ and executes the following instructions:

1. computes the sum of the elements below the main diagonal (excluding the elements along the diagonal)

2. computes the maximum of the elements upper the main diagonal (excluding the elements along the diagonal)
3. computes the minimum of the elements along the main diagonal
4. outputs all the results

Exercise 24. Write an algorithm that reads a matrix of reals $\mathbf{X}_{I\times J}$ and computes the following quantities:

1. $a = \sum_{i=1}^{I} \sum_{j=1}^{J} \mathbf{X}_{i,j}$ with $i \neq j$
2. $b = \sum_{l\in\mathcal{L}} \sum_{h\in\mathcal{H}} \mathbf{X}_{l,h}$ with $\mathcal{L}$ being the set of *even* indices and $\mathcal{H}$ the set of *odd* indices
3. computes $q = a + b$ if $a > b$, otherwise $q = \frac{1}{2}(a-b)^2$
4. outputs q

Exercise 25. Write an algorithm that reads two numbers $N > 0$, K and finds a square matrix $\mathbf{A}_{N\times N}$ such that the sum of elements in every row and column is equal to K.

Exercise 26. Write an algorithm that reads two numbers $N > 0$, K and finds a square matrix $\mathbf{A}_{N\times N}$ such that

$$\sum_i \mathrm{diag}(\mathbf{A})_i = K\,(\max(\mathbf{A}) - \min(\mathbf{A}))$$

Exercise 27. Write an algorithm that reads a positive integer I and two vectors of *different* integers $\mathbf{n}_{I\times 1}$, $\mathbf{m}_{I\times 1}$, and finds a matrix $\mathbf{A}_{I\times I}$ such that

$$a_{i,j} = n_i + m_j$$

for all $i, j \in \{1, \dots, I\}$.

Exercise 28. Write an algorithm that reads a vector of characters $\mathbf{s}_{K\times 1}$ and determines the matrix $\mathbf{S}_{N\times M}$ filled by all the characters containing in $\mathbf{s}$, where $N = \lfloor\sqrt{K}\rfloor$ and $M = \lceil\sqrt{K}\rceil$.
TIPS: The values of $\lfloor a\rfloor$ and $\lceil a\rceil$ can be computed using the primitive function `floor()` and `ceil()`, respectively.

Exercise 29. Write an algorithm that reads a matrix of reals $\mathbf{X}_{N\times N}$ and an integer N, and sorts $\mathrm{diag}(\mathbf{X})$ in descending order.

Exercise 30. Write an algorithm that reads two matrices of the same order $\mathbf{A}_{N\times N}$, $\mathbf{B}_{N\times N}$, and defines a new matrix $\mathbf{C}_{N\times N}$ containing only corresponding common elements and place `NA` at the positions where elements are different.